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Self-Compensation for the Axial Velocity Spread in a Wiggler Field

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Abstract. In order to obtain optimal performance from a free electron laser (FEL), the axial velocity spread on the electron beam must be small as it propagates through the wiggler field. Treated separately, both the wiggler-induced betatron motions and the self-induced space charge forces tend to increase the axial velocity spread and degrade the performance of the FEL. However, it has been shown analytically [B. Hafizi and C.W. Roberson, Phys. Plasmas **3**, 2156 (1996)] that when both effects are treated self-consistently an equilibrium exists wherein the space charge forces exactly compensate for the betatron motion. This leads to the surprising result that for a continuous beam, increasing the beam current can improve the beam quality.

INTRODUCTION

The quality of the electron beam used in a free electron laser (FEL) can be quantified in terms of the scaled thermal velocity, S, given by [1]

$$S = \frac{v_{th,z}}{v_b - v_{ph}}$$

where v_b is the average velocity of the beam, $v_{th,z}$ is the root-mean-square (rms) spread in the axial velocity, and v_{ph} is the phase velocity of the fastest growing ponderomotive wave. When the axial velocity spread is large enough so that $S \gg 1$, the FEL operates in a kinetic regime where the cold beam approximation is invalid. This generally results in a drastic reduction in the efficiency of the FEL since only a fraction of the electrons contribute to the growth of the radiation.

The axial velocity spread on the beam is initially determined by the beam emittance and energy spread. As the beam propagates, both the wiggler fields and the beam's own self-fields cause the axial velocity spread to evolve. When the beam current is low, the self-fields can be neglected, and the axial velocity spread only evolves because of the betatron oscillations caused by the wiggler field. This leads to an axial velocity spread in the form of a shear $\nabla_\perp v_z$. Here, v_z is the axial velocity of the electron fluid and $\nabla_\perp$ is the transverse gradient. When the self-fields are included, they oppose the betatron oscillations and reduce the shear. Indeed, in the case of a continuous beam, an equilibrium can be found such that forces due to the self-fields exactly cancel the forces due to the focusing fields of the wiggler [1]. In this paper, we perform numerical

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calculations illustrating this equilibrium. The case of a pulsed beam will be considered in a future paper.

NUMERICAL MODEL

We study the electron beam flow in a FEL by numerically following the orbits of a large number of particles under the influence of a prescribed wiggler field and a dynamically computed self-field.

The particles are initially loaded into a Gaussian weighted hyper-ellipsoid in six dimensional phase space. They are advanced in time using the standard Boris pusher [2].

The self field model is derived from a method used in the Los Alamos code Trace3D. First, an ellipsoid is fitted to the particle positions via

$$\sigma_\alpha^2 = \frac{1}{N}\sum_i (\alpha_i - C_\alpha)^2$$

where N is the number of particles, i is the particle index, α is one of the three spatial coordinates, and C_α are the coordinates of the beam centroid. The force exerted on a particle by the self fields is then approximated by

$$F_x = 5^{-3/2}\frac{q}{4\pi\varepsilon_0}\frac{3Q}{\gamma^2}\beta\frac{1-f}{\sigma_x(\sigma_x+\sigma_y)\sigma_z}x \tag{1}$$

$$F_y = 5^{-3/2}\frac{q}{4\pi\varepsilon_0}\frac{3Q}{\gamma^2}\beta\frac{1-f}{\sigma_y(\sigma_x+\sigma_y)\sigma_z}y \tag{2}$$

$$F_z = 5^{-3/2}\frac{q}{4\pi\varepsilon_0}3Q\beta\frac{f}{\sigma_x\sigma_y\sigma_z}z \tag{3}$$

where ε_0 is the permittivity of free space, q is the charge of the particle, Q is the total charge in the beam, γ is the relativistic factor of the beam, and f is the "space charge form factor" which was tabulated by Lapostolle using data obtained numerically. In this paper, $\sigma_z \to \infty$ which leads to $f \to 0$. The factor of $5^{-3/2}$ converts the dimensions of the Gaussian weighted ellipsoid into the dimensions of the equivalent uniform weighted ellipsoid.

The wiggler field is modeled according to the formula given in Ref. [3] for parabolic pole pieces:

$$\begin{aligned} B \;=\; B_0\Big[&\sinh(2^{-1/2}k_w x)\sinh(2^{-1/2}k_w y)\cos(k_w z)\hat{\mathbf{x}} \\ &+\cosh(2^{-1/2}k_w x)\cosh(2^{-1/2}k_w y)\cos(k_w z)\hat{\mathbf{y}} \\ &-2^{-1/2}\cosh(2^{-1/2}k_w x)\sinh(2^{-1/2}k_w y)\sin(k_w z)\hat{\mathbf{z}}\Big] \end{aligned}$$

Here, k_w is the wiggler wavenumber and $\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}}$ are the unit vectors. The electron beam propagates in the z-direction, with the wiggle motion in the x-z plane. The parabolic pole pieces provide equal focusing in both transverse directions.

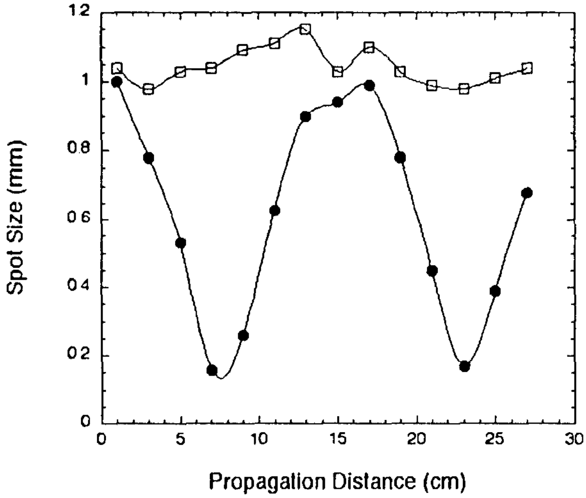

FIGURE 1. Variation of spot size with propagation distance for a beam with and without space charge. Solid circles represent data from a calculation with $J = 0$, open squares represent data from a calculation with $J = 8$ kA/cm^2. As expected, self fields extend the betatron wavelength.

SELF COMPENSATION

In the case of a continuous beam, the effect of self fields is to modify the betatron wavenumber according to [1]

$$k_\beta = k_{\beta 0}(1 - \mathrm{SFP})^{1/2} \tag{4}$$

where

$$\mathrm{SFP} = \left(\frac{k_p/k_{\beta 0}}{\gamma_{z0}\beta_{z0}\gamma_0^{1/2}}\right)^2$$

Here, $k_p^2 = 4\pi ne^2/mc^2$, n is the electron density, e is the electronic charge, $k_{\beta 0} = a_w k_w/\sqrt{2}\gamma_0\beta_{z0}$ is the betatron wavenumber due to the wiggler field only, $a_w = eB_0/k_w mc^2$ is the peak normalized vector potential of the wiggler field, β_{z0} is the axial velocity of the beam normalized to c, and $\gamma_{z0} = (1 - \beta_{z0}^2)^{-1/2}$. Although this formula was derived for flat pole pieces, it remains valid for parabolic pole pieces as well. Note that if the self field parameter (SFP) is unity, the betatron wavenumber vanishes and the beam is in equilibrium.

In the case of finite emittance, the matched beam condition becomes

$$\varepsilon_n = \gamma_0\beta_{z0}k_\beta\sigma_r^2$$

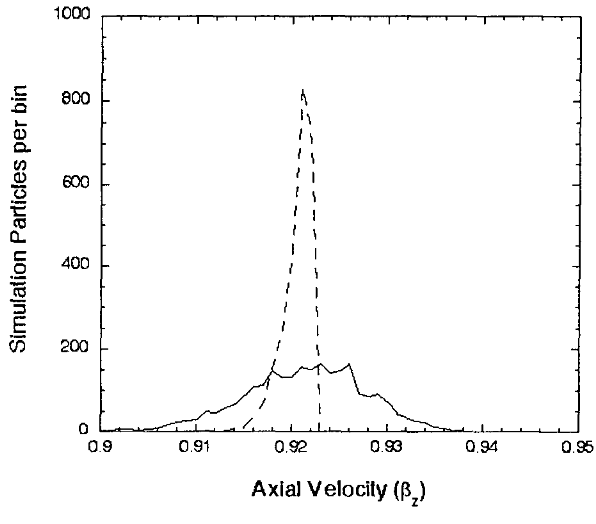

FIGURE 2. Axial velocity distribution for a continuous beam 10 cm into the wiggler field. The solid line is the data for $J = 0$, the dashed line is the data for $J = 8$ kA/cm^2

where ε_n is the normalized emittance (in units of length · radians), and σ_r is the radius of a uniform density continuous beam. For a finite spot size, equilibrium occurs for $k_\beta > 0$. However, for emittances typical of a high quality photo-injector [4, 5], the beam is nearly in equilibrium for $k_\beta = 0$.

We now consider numerical calculations of the electron orbits in a wiggler field with $B_0 = 3$ kG and $\lambda_w = 2\pi/k_w = 5$ cm. The beam had initial dimensions of $\sigma_x = \sigma_y = 1$ mm, and initial emittance of zero. Fig. 1 shows measurements of σ_y at several z positions. The solid circles correspond to the case where the current density, J, vanishes. In this case the beam undergoes betatron oscillations at the expected frequency. The open squares correspond the case where $J = 8$ kA/cm^2 which gives SFP $= 0.8$. In this case the spot size is nearly constant. According to Eq. (4), the best equilibrium should occur for SFP $= 1$. In the numerical calculation, however, SFP $= 1$ led to a larger deviation in spot size than SFP $= 0.8$. The discrepancy may be related to the conversion between Gaussian and uniform beams.

Fig. 2 illustrates the effect of space charge on the axial velocity distribution. The solid curve corresponds to the case where $J = 0$ while the dashed curve corresponds to the case where $J = 8$ kA/cm^2. The distribution is significantly narrower in the case of the high current beam. This is a result of the fact that in the presence of strong self-fields forward momentum is not lost to betatron motions.

CONCLUSIONS

Numerical calculations support the assertion that self-fields can improve the quality of an electron beam for FEL applications. By opposing the focusing forces of the wiggler field, self-fields can increase the betatron wavelength and reduce the spread in axial velocity. This reduces S and improves the performance of the FEL.

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